# Table of Contents

**Abstract** **ix**

**Deutsche Kurzfassung** **xi**

**1 Introduction** **1**
1.1 Motivation and Focus . . . 1
1.2 Contributions and Organization . . . 3

**2 Polyhedral Approximation Methods for Cooperative Optimization** **11**
2.1 Introduction . . . 11
2.2 Distributed Algorithms in Peer-to-Peer Networks . . . 14
2.2.1 Communication Network Model . . . 14
2.2.2 Distributed Algorithms . . . 15
2.2.3 Complexity Notions . . . 16
2.3 The Cutting-Plane Consensus Algorithm . . . 17
2.3.1 General Problem Formulation . . . 18
2.3.2 Unique Solution Linear Programming . . . 20
2.3.3 The Algorithm Definition . . . 26
2.3.4 Technical Analysis . . . 27
2.4 Convex Inequality Constraints . . . 32
2.4.1 Problem Formulation . . . 32
2.4.2 Semidefinite Constraints . . . 33
2.4.3 Linear Constraints . . . 34
2.4.4 Application Example: Position Estimation in Wireless Sensor Networks 36
2.5 Robust Optimization with Uncertain Constraints . . . 38
2.5.1 Problem Formulation . . . 38
2.5.2 Efficiently Solvable Problems . . . 40
2.5.3 Computational Study: Robust Linear Programming . . . 41
2.6 Conclusions . . . 43

**3 Dual Cutting-Plane and Trajectory Exchange Optimization** **45**
3.1 Introduction . . . 45
3.2 A Motivating Problem: Distributed Cooperative Model Predictive Control 46
3.2.1 Problem Formulation . . . 46
3.2.2 Dual Semi-Infinite Problem Representation . . . 48
3.3 Revisiting the Richards and How Algorithm . . . 49
3.4 Distributed Nonlinear Dantzig-Wolfe Decomposition . . . 53
3.4.1 Distributed Constraint Generation . . . 53
3.4.2 Linear Programming Dual Interpretation . . . 55
3.4.3 CPC-based Trajectory Exchange Method . . . 58

3.5 Application Example: Distributed Microgrid Control . . . 59
3.6 Conclusions . . . 63

**4 Duality and Network Theory in Cooperative Control 65**
4.1 Introduction . . . 65
4.2 Preliminaries . . . 67
4.2.1 Algebraic Graph Theory . . . 67
4.2.2 Network Theory . . . 70
4.2.3 Equilibrium Independent Passivity . . . 72
4.3 Duality in Passivity-based Cooperative Control . . . 75
4.3.1 The Plant Level . . . 77
4.3.2 The Control Level . . . 80
4.3.3 The Closed-Loop Perspective . . . 82
4.4 Application Example: Optimal Distribution Control . . . 85
4.5 Conclusions . . . 87

**5 Clustering in Dynamical Networks 89**
5.1 Introduction . . . 89
5.2 Constrained Flows & Network Clustering . . . 91
5.2.1 A Primal/Dual and Saddle-Point Perspective . . . 91
5.2.2 Saddle-Point Problem and Network Clustering . . . 93
5.3 Clustering in Dynamical Networks . . . 95
5.3.1 A Dynamical Model for Clustering . . . 96
5.3.2 Clustering Analysis and Convergence . . . 98
5.3.3 Application Examples . . . 100
5.4 Hierarchical Clustering Using a Saddle-Point Analysis . . . 106
5.4.1 Combinatorial Conditions for Clustering . . . 106
5.4.2 A Hierarchical Clustering Algorithm . . . 109
5.4.3 Application Example: Structural Analysis of Power Networks . . . 111
5.5 Conclusions . . . 114

**6 Conclusions and Outlook 117**
6.1 Conclusions . . . 117
6.2 Outlook . . . 119

**A Convex Analysis and Optimization Theory 121**

**B Dynamical Systems and Control Theory 127**

**C Graph Theory 131**

**Bibliography 135**